When education is not enough: Explaining labor informality inertia in Chile

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Abstract

This paper studies the evolution of labor informality in Chile, with a twofold contribution. First, since the country only has an official measure of informality from 2017, I propose a way of measuring informality in which the formal share of employment is consistent with administrative records. Then, I generate a series of informality from 1990 to 2020, which is surprisingly stable. This is counter-intuitive since Chile has experienced a significant increase in tertiary education in the last three decades, a component often linked to informality reduction. Second, I adapt a search and matching model that explains the decrease in labor informality in Brazil to the Chilean case, and I estimate it using data from 2006-2017. The model is focused on the general equilibrium effects that affect informality when the skill composition of the workforce changes. I find that increases in the real minimum wage and the decrease in the TFP offset the impact of rising tertiary education, contributing to the observed stability in informality levels. The above sheds light on the differences between the Brazilian and Chilean economies, highlighting potential diminishing returns in the composition effects mentioned, and structural differences in the informal sector. Lastly, I delve into the broader implications of these findings, with a focus on pertinent policies that could be implemented to alleviate informality in the country.

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1 Introduction

Labor informality presents a pressing concern within developing economies, with far-reaching consequences for social welfare. In these contexts, labor regulations do not extend to informal workers, exempting them from minimum wage requirements and regulated working hours. Furthermore, the informal sector generally evades taxation, which can strain government finances. Informal workers and firms also tend to sidestep contributions to the social security system, placing additional financial stress on health and pension programs. The magnitude of these challenges is substantial, with labor informality affecting a significant portion of the workforce, ranging from 20% to 80% in some developing countries (Perry et al., 2007; La Porta & Shleifer, 2014). The informal sector's contribution to GDP is also noteworthy, accounting for nearly one-third of the total economic activity (Schneider & Enste, 2000; Elgin et al., 2021).

Previous literature has extensively examined the significance of labor informality, beginning with the fundamental question of whether informality is inherently problematic. It has been proposed that minimum wage and labor regulations may inadvertently serve as barriers to entry for less productive workers (Magnac, 1991). In such a scenario, informality could be viewed as a preferable alternative to unemployment. In this context, increasing government oversight might reduce informality but potentially increase unemployment. Thus, labor informality may represent a middle ground between formal employment and unemployment, prompting policymakers to consider a nuanced approach rather than striving for its complete elimination.

The empirical literature generally leans towards the notion that reducing labor informality is welfare-improving. However, the assessment of this claim depends on various mechanisms, and it requires careful examination. For instance, Almeida & Carneiro (2012) argue that increasing government oversight with a focus on enforcing mandated benefits in formal firms could enhance welfare, as evidenced in the case of Brazil. The underlying mechanism here suggests that heightened government enforcement leads to lower wages in the formal sector, as firms may reduce wages to cover the costs of providing benefits.[1] However, it is important to note that formal firms subject to a binding minimum wage cannot decrease wages, which can result in improved conditions for formal workers earning around that wage. This, in turn, could incentivize low-skilled workers to transition from the informal to the formal sector. Consequently, the informal sector may need to raise wages to attract labor, potentially expanding its size. While Cardoso & Lage (2007) identified a focus on enforcing mandated benefits in formal firms by labor inspectors in Brazil, it is essential to acknowledge that these findings may not generalize to other countries.

[1] This concept was previously explored by Almeida & Carneiro (2009), who found that stricter enforcement of labor regulations constrained the size of formal firms due to reduced wages.

The examination of labor market frictions has spurred the development of search models in the literature, shedding light on how these frictions influence labor informality. A common argument in this context posits that labor informality becomes problematic in the presence of search frictions. When both formal and informal firms compete for the same labor market to find workers, informal firms create a negative externality for formal firms by reducing the likelihood of finding suitable workers. Given that informal firms often exhibit characteristics such as smaller size, lower wage levels, a preference for less-educated workers, and offer less stable jobs (Perry et al., 2007; La Porta & Shleifer, 2008; De Paula & Scheinkman, 2011; La Porta & Shleifer, 2014), policies that enhance the mandated benefits for formal workers may incentivize labor migration from the informal to the formal sector. However, Albrecht et al. (2009) demonstrated that such policies might be perceived as increasing the costs associated with operating a formal firm, potentially leading to reduced formal employment and, in the broader context, higher unemployment rates.

Another significant aspect of the labor informality discussion revolves around the question of whether formal and informal firms share the same labor market or operate in separate spheres, a concept commonly referred to as the "dualistic view" of the labor market (Ozorio et al., 1995; Maloney, 1999). An illustrative instance of this research can be found in Ulyssea (2010), where a model is introduced featuring distinct formal and informal markets and workers randomly search for job opportunities. This modelling approach offers the advantage of calculating separate measures of tightness, vacancy rates, and employment probabilities for each sector. The author calibrated this model using Brazilian data and demonstrated that the costs associated with entering the formal sector play a pivotal role in determining the size of the informal sector. This finding holds particular significance, as it suggests that by focusing policy efforts on reducing entry costs into the formal sector, policymakers can effectively address the trade-off between lower informality and higher unemployment rates. Additionally, this type of model has the capacity to explain the observed compensating differentials between workers in both sectors, especially concerning highly educated workers in the informal sector (Fields, 1990; Rauch, 1991; Günther & Launov, 2012).

Although the notion of reducing entry costs for formal firms gained widespread acceptance in subsequent literature, empirical evidence did not support the idea of distinct, separated labor markets. Initially, Maloney (1999) presented evidence indicating that worker transitions between the formal and informal sectors did not imply the existence of two separate labor markets. Subsequently, studies conducted by Meghir et al. (2015), Allen et al. (2018), and Ulyssea (2018) found that productivity distributions for formal and informal firms overlapped in both Brazil and India. Since then, the prevailing approach in most scholarly work has been to model formal and informal jobs within the same labor market. Characteristics such as educational levels have been utilized to differentiate between the two sectors.

On the contrary, actions that raise the operating costs of informal firms within an economy can potentially be welfare-improving. Such actions facilitate the reallocation of workers from the informal to the formal sector, yielding associated benefits. This is feasible due to the presence of compensating differentials among workers with similar observable characteristics. Studies by Bosch & Esteban-Pretel (2012), Charlot et al. (2013), and Meghir et al. (2015) have demonstrated the viability of this channel when firms must decide whether to operate formally or informally, a decision referred to as the extensive margin of informality. Expanding on this notion, Ulyssea (2018) extended the model to encompass the intensive margin, wherein formal firms may also hire workers informally. The author's findings indicated that incorporating the intensive margin weakened the conclusion that reducing informality directly leads to higher economic output. The inclusion of both margins is significant, as existing literature suggests that between one-third and one-half of informal employment takes place within formal firms (Samaniego de la Parra, 2017; Ulyssea, 2018; Cisneros-Acevedo, 2022). Subsequently, Ulyssea (2020) concluded that the most effective approach to address informality in a welfare-improving manner is by enhancing government enforcement at the extensive margin.

Another cluster of studies examines the impact of minimum wage policies on labor informality. Boeri et al. (2011), utilizing Brazilian data, showed that increases in the minimum wage result in higher wages within the informal sector. They attributed roughly one-third of the total effect to a sorting effect, where workers transition from the formal to the informal sector in response to a minimum wage hike. In a similar vein, Comola & De Mello (2011) observed a comparable effect in Indonesia. They found that an increase in the ratio of the minimum wage to the mean wage led to a net increase in employment. This was primarily due to the rise in employment within the informal sector, which more than offset the decline in formal sector employment. However, it is worth noting that the impact of these changes varies across sectors. Since the minimum wage exerts a stronger influence on certain industries than others, the aggregate effects are not uniform. As Parente (2022) pointed out, policies aimed at reducing income inequality, such as minimum wage increases, can inadvertently contribute to greater inequality by shifting workers into the informal sector.

The recognition that policies can have divergent effects on individuals has motivated research into various factors that influence labor informality, encompassing trade dynamics (Dix-Carneiro et al., 2021), financial frictions (Flabbi & Tejada, 2022), corporate taxation (Rocha et al., 2018; Cisneros-Acevedo & Ruggieri, 2022), and human capital accumulation (Bobba et al., 2021). In this context, one critical determinant of heterogeneous effects is education. Previous studies either assumed homogeneous workers or exogenous levels of education. Consequently, a pertinent question arises: how does the level of education attained by workers respond to the prevalence of informality in the labor market? Bobba et al. (2022) address this inquiry by constructing a model

where educational levels are endogenous. Their findings suggest that in such a framework, the presence of labor informality can be welfare-improving, as it creates employment opportunities for workers with lower levels of education (which is costly to acquire in the model). However, this comes at the cost of reducing overall levels of schooling.

However, it's worth noting that the aforementioned study exclusively examines the effects of education in terms of levels without considering broader general equilibrium effects. To address this gap, Haanwinckel & Soares (2021) conducted an investigation into how the changes in education levels experienced by Brazil between 2000 and 2010 impacted the prevalence of labor informality. Their primary finding is that alterations in the skill composition, specifically an increase in the proportion of high-skilled workers, corresponded to a reduction in informality levels. The proposed mechanism hinges on the notion of complementarity across skill types. Even as firms continue to require low-skilled workers for production, there is a heightened scarcity of such workers. Consequently, firms find themselves in competition for low-skilled labor. Given the greater productivity of formal firms, they have the capacity to attract workers from the informal sector, ultimately driving up wages and encouraging the transition to formal employment among low-skilled workers.

A consistent trend observed across studies focusing on education and labor informality is that individuals with higher levels of education tend to exhibit a greater propensity for formal sector employment. In the case of Bobba et al. (2022), individuals with more education not only display higher productivity, making them more attractive to formal sector employers, but they also experience a higher rate of job offers. In Haanwinckel & Soares (2021), the value of more educated workers is emphasized, particularly for high-productivity firms, where it is assumed that the marginal productivity of such workers increases with firm productivity. Consequently, an increase in the proportion of high-skill workers, all else being equal, leads to a mechanical reduction in labor informality. Furthermore, depending on the composition effects, this shift can potentially contribute to a reduction in unemployment rates.

According to the findings of Haanwinckel & Soares (2021), the effect of shifts in the proportion of high-skill workers is so substantial that it can offset measures that would typically exert pressure to increase the prevalence of labor informality. For instance, during the period of 2003-2012 in Brazil, the GDP per capita increased by 31%, while the minimum wage surged by 61%, thereby creating considerable pressure for informality to rise. However, concurrently, the share of workers with tertiary education also increased significantly, rising from 4.4% to 11%. The authors assert that this educational shift played a pivotal role, resulting in a reduction in informality from 28% to 17% during the same period. Furthermore, unemployment rates decreased from 13.4% to 5.9%, as a substantial portion of low-skilled workers successfully found formal employment.

Hence, nations witnessing substantial advancements in higher education levels can anticipate a decline in informality rates, provided that increases in the minimum wage do not significantly lag behind rises in GDP per capita. The case of Chile serves as a relevant illustration. Over the period from 2006 to 2017, Chile recorded a 36% increase in the minimum wage, coupled with a 27% growth in GDP per capita. Furthermore, due to concerted efforts in public policy aimed at expanding tertiary education access, the country witnessed an increase in the proportion of individuals pursuing higher education, rising from 27% to 38%.

It is crucial to note that Chile lacks official data on labor informality for the period prior to 2017. To address this data gap, I relied on the primary household survey and administrative records of formal workers to construct an informality measure spanning from 1990 to 2020. This definition aligns with established criteria in previous literature and remains compatible with the officially reported rates from 2017 onwards. Within this context, it might have been plausible to anticipate a reduction in the informal sector's share in 2017; however, informality exhibited minimal change, decreasing by just one percentage point compared to the 2006 rate (from 33% to 32%).

In this paper, I posit that while general equilibrium effects play a significant role, their impact varies substantially based on the pre-existing levels of tertiary education within economies. This impact tends to diminish as economies witness higher levels of tertiary education. This perspective offers a potential explanation for the dynamics observed in both the Brazilian and Chilean cases.

To achieve these objectives, I adapt the model introduced by Haanwinckel & Soares (2021) and calibrate it using data specific to Chile. For the estimation phase, I will closely follow the original model proposed by these authors. However, in the validation exercise, I will introduce an alternative method, as the original approach is not consistent with the Chilean context. Through this process, I aim to demonstrate that the adapted model accurately predicts informality levels in 2006 while maintaining a reasonable level of fit for the rest of the variables. Furthermore, in the validation exercise, I will illustrate how the model can account for the stability of informality following the policy changes implemented in Chile between 2006 and 2017.

Finally, it is essential to note that I have chosen to utilize the Haanwinckel & Soares (2021) model as the foundational framework for this study due to its incorporation of critical features not found in alternative models. Haanwinckel & Soares (2021) built upon the intra-firm bargaining theory initially proposed by Stole & Zwiebel (1996) and further developed by Cahuc et al. (2008). They extended this framework to encompass both formal and informal sectors, making it particularly relevant to the study of labor informality. Moreover, the model presented by Haanwinckel & Soares (2021) incorporates labor regulations, such as minimum wage and payroll taxes, mirroring elements found in the work of Acemoglu & Hawkins (2014). As such, the model devised by

Haanwinckel & Soares (2021) represents a noteworthy advancement in the understanding of labor informality, offering valuable insights into the contemporary frontiers of research in this field.

This paper is organized as follows: The second section provides an overview of the data sets used and the proposed methodology for measuring labor informality. The third section presents descriptive statistics on the Chilean labor market, offering insights into the transformations it has undergone over the past three decades, particularly in terms of tertiary education. The fourth section introduces the baseline model while discussing its properties and limitations. In the fifth section, I present the estimation results specific to Chile, discuss the validation exercise, and conduct the counter-factual exercises. The sixth section delves into the broader implications of the results, highlighting the differences between the Brazilian and Chilean economies. Finally, the seventh section concludes the paper, summarizing the key findings and their implications.

2 Measuring labor informality

To construct the informality series, I utilized data from two primary sources. The first source is the CASEN survey, a comprehensive cross-sectional household survey conducted in Chile. Spanning 14 waves from 1990 to 2020, this survey collects a wide range of information, including gender, labor outcomes, education levels, access to social security, and income, making it a valuable resource for characterizing workers in both the formal and informal sectors.

The second source comprises administrative data obtained from the unemployment insurance management agency (SC).[2] This dataset comprises samples from 3%, 5%, and 12% of workers affiliated with unemployment insurance. Notably, the SC data exclusively includes information related to what can be categorized as formal workers.

Generating a measure of labor informality in Chile presents challenges due to the absence of an official definition of formal employment. In contrast, some countries, such as Brazil, offer a more straightforward criterion for formal employment: the employer's signature on the worker's labor card (*carteira de trabalho*). This signed card serves as a useful indicator of labor formality, as it grants employees access to social security benefits and obligates employers to adhere to minimum wage legislation while fulfilling their social security contributions and payroll tax obligations (Meghir et al., 2015; Haanwinckel & Soares, 2021).

In Chile, the absence of a universal labor card and the segmented nature of social security contributions complicate the definition of informality. Social security contributions in Chile are divided

[2] *Administradora del Fondo de Cesantía* (AFC) in Spanish, is a private institution that administrates the unemployment funds in Chile since 2002.

into three categories, each corresponding to a different proportion of the worker's wage: pensions (10%), health (7%), and unemployment insurance (0.6%). Notably, workers or employers can be responsible for contributing to pensions and health, depending on the employment arrangement, while unemployment insurance contributions are exclusive to formal employers.[3] Consequently, defining informality necessitates choosing one or more of these social security payments.

Given the complexities of Chile's social security system, utilizing payment of unemployment insurance could provide a robust definition of formal employment that aligns closely with administrative records. However, a significant challenge arises as the CASEN survey does not consistently inquire about unemployment insurance payments, except in the 2006 and 2009 waves, where approximately 16% and 13% of workers did not know, respectively. Subsequently, this question was omitted in subsequent waves. To address this limitation, I propose an alternative definition, classifying a worker as formally employed if their employing firm contributes to the pension system. It is worth noting that, according to the Chilean labor law, any firm making pension contributions is also obligated to contribute to the unemployment insurance, creating a strong correlation between the two variables. Nonetheless, some exceptions exist, such as firms contributing to pension and health systems for owners working within the company but exempting them from unemployment insurance payments. Consequently, this definition of labor formality excludes self-employed workers. Another exception applies to certain public sector workers, especially those in central government administration, where the employer does not cover social security contributions, and those who work in the armed forces and the police (who have different pension and health systems).

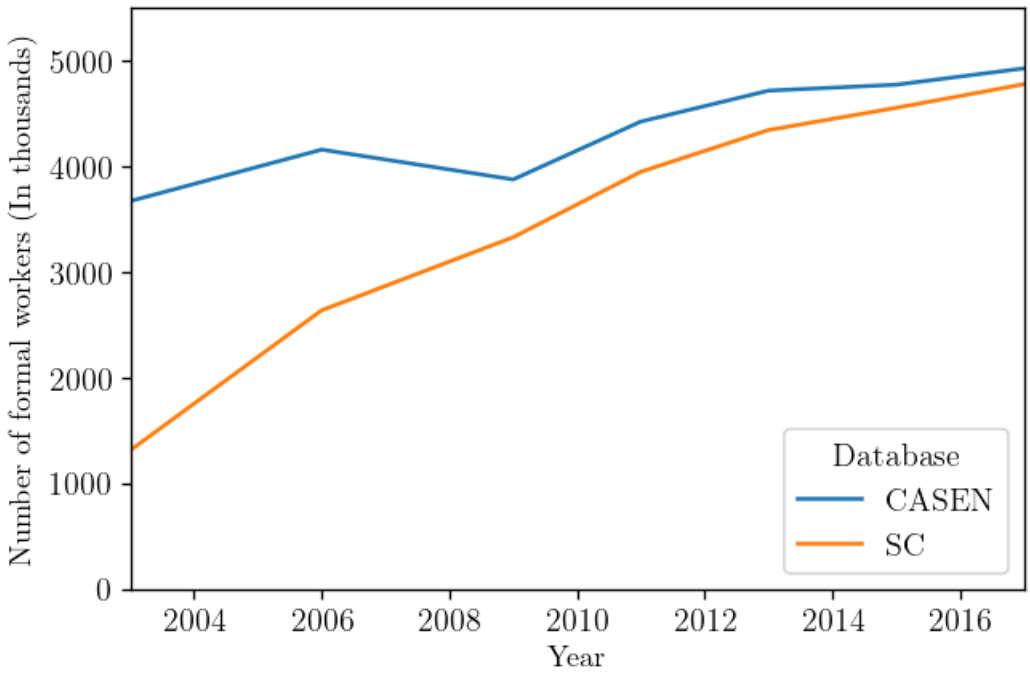
However, even if these variables exhibit a strong correlation, it remains uncertain *a priori* whether individuals who declare that their employers are paying pension contributions are indeed doing so. Recognizing this potential bias, it becomes essential to validate the correlation between these variables using an alternative database.

To address this, I will calculate the number of individuals who would be considered formally employed according to the CASEN survey, using the previously mentioned definition. I will then compare this number with the count of individuals affiliated with the unemployment insurance, as documented in administrative records during the same period. If these values closely align, it provides confidence in the reasonability of the informality definition. With this validated definition, I can consistently apply the same question across all waves of the CASEN survey, enabling the construction of a labor informality series spanning from 1990 to 2020.

[3]It is worth noting that government administration workers (excluding those in public firms) do not have access to unemployment insurance coverage. Additionally, the maximum income considered for these contributions is approximately equivalent to around the 95th percentile of labor income, rendering it largely irrelevant for the majority of workers.

However, another challenge arises from the introduction of unemployment insurance in Chile in 2002. While this insurance became mandatory for newly hired formal workers, it remained optional for employees who did not change their employer. Consequently, there is a transitional period during which the number of workers affiliated with unemployment insurance may not accurately represent the count of formal workers.

Figure 1: Evolution of formal workers according CASEN and SC datasets



Source: Data from CASEN surveys correspond to waves 2003, 2006, 2009, 2011, 2013, 2015 and 2017. To construct the series in CASEN I am using the frequency weights to compute the population. Data from Unemployment Insurance (SC) corresponds to number of affiliates in December of the years mentioned above.

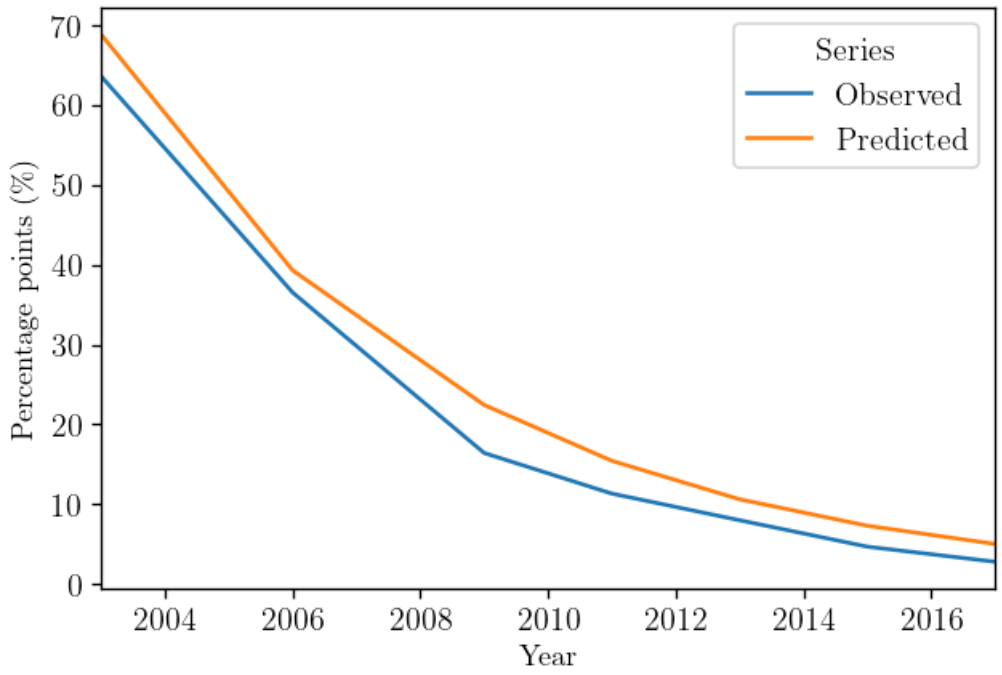
To illustrate this, Figure 1 displays the trends in formal workers using the proposed definition based on CASEN data and the evolution of workers covered by unemployment insurance. Notably, the CASEN series exhibits a slower growth rate compared to the SC series, which aligns with the factors mentioned earlier. Furthermore, both series ultimately converge, providing strong support for the validity of the informality definition.

A robustness check for this measure involves comparing the duration of a job implied by the convergence rate with the one reported in the CASEN survey. For workers formally employed before the introduction of the unemployment insurance system, they only appear as covered by insurance once they move to another formal job. Consequently, the average duration of formal jobs governs the convergence of both series.

Using data from the employment survey in 2011, which falls in the middle of the studied period, I find that the average duration of a formal job is approximately 5.35 years. This average is used to simulate an exponential distribution with the parameter $\lambda = 0.19$ (the inverse of the average duration, i.e., 1/5.35).

Figure 2 illustrates how the SC series should evolve if individuals who were formally employed before the introduction of unemployment insurance eventually leave their jobs, whether due to layoffs or voluntary quits, and subsequently secure another formal job. While this assumption is admittedly strong, the close alignment between the observed and predicted differences suggests that the definition of a formal job based on the CASEN survey is consistent with the movement observed in the SC dataset.

Figure 2: Observed and predicted difference between formal jobs according to SC and CASEN datasets

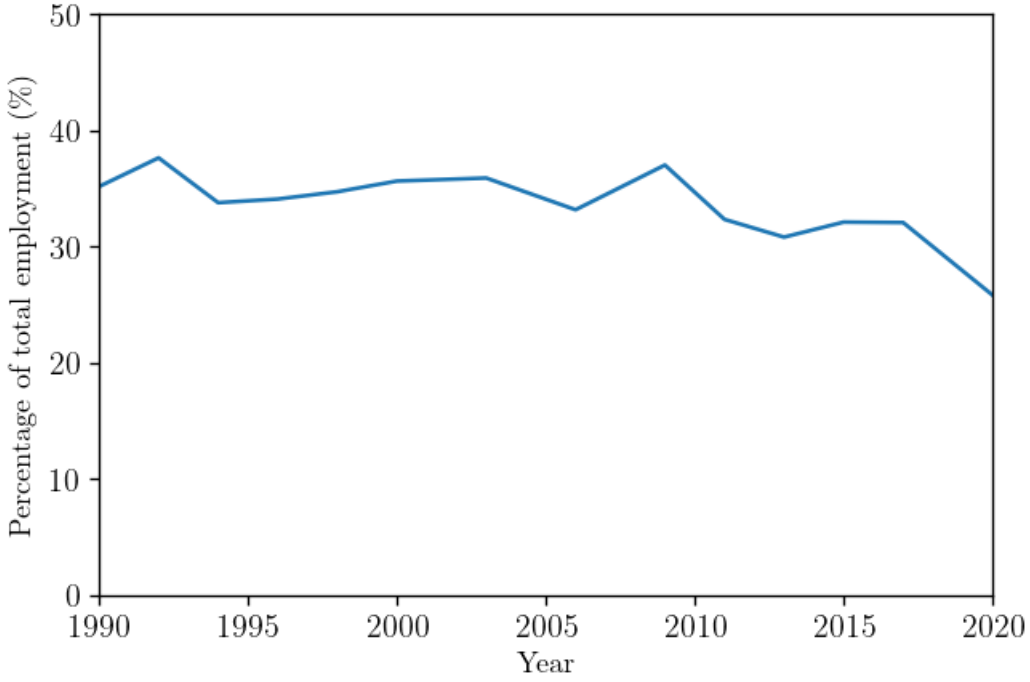


Source: Own elaboration using percentage difference of the series presented in Figure 1 and the predicted difference using an exponential distribution with parameter $\lambda = 0.19$.

Hence, it is feasible to construct a labor informality series for Chile using the proposed definition for each wave of the CASEN surveys. Figure 3 displays the evolution of labor informality in the country from 1990 to 2020. Notably, the informality rate has remained relatively stable, fluctuating within the range of 30-40% over the past three decades. Two significant events stand out in the series: the Great Recession of 2008-2009 and the pandemic-induced crisis of 2020. Interestingly, these crises had contrasting effects on labor informality; it increased during the first

and decreased during the second.

Figure 3: Evolution of labor informality in Chile in the period 1990-2020.



Source: Own elaboration using CASEN surveys for the period 1990-2020.

Finally, it is noteworthy to compare this measure with the official labor informality series produced by the National Bureau of Statistics of Chile (INE). Since 2017, INE's labor department has computed informality rates using a stringent definition that considers two sources of informality: one related to workers (e.g., absence of a contract, non-payment of social security contributions) and another associated with employers (e.g., lack of registration at the tax office, non-payment of operating permits). Unfortunately, this stricter definition cannot be applied to years prior to 2017 due to the absence of certain survey questions in earlier years. Nonetheless, the disparities between the two series are relatively small. The official series reported informality rates of 30% in 2017 and 27% in 2020, while my proposed definition indicates rates of 32% in 2017 and 26% in 2020. This modest difference suggests that, in Chile, the predominant form of labor informality is driven by the extensive margin, primarily involving informal firms offering informal employment opportunities.

3 Descriptive statistics

This section will present some descriptive statistics that can be useful in understanding the puzzling situation in Chile regarding labor informality. Before presenting the summarized statistics,

it is important to briefly outline the filtering criteria that will be applied in their computation

To ensure data consistency, the initial dataset was filtered to include individuals falling within the age range of 18 to 65 for men and 18 to 60 for women. This distinction accounts for variations in retirement ages in Chile. Furthermore, the dataset encompasses individuals residing in both urban and rural areas across all regions of the country. Inactive individuals were excluded from the calculations to maintain the focus on the active labor force.

As Haanwinckel & Soares (2021) utilizes data from 2003 to 2012 to estimate the model and its implications, I have chosen to focus on the period between 2006 and 2017 for this analysis. These years were selected because they are nearly a decade apart and encompass the Great Recession, then it could be considered analogous to the period examined in the baseline paper. Table 1 provides a summary of labor market statistics in Chile for the years 2006 and 2017.

Table 1: Summary statistics: Chilean labor market (2006-2017)

	Wages in 2006		Wages in 2017		Informality (%)		Unemp. (%)		Shares	
Sample	Formal	Informal	Formal	Informal	2006	2017	2006	2017	2006	2017
All	2.20	1.26	2.16	1.44	0.33	0.32	0.07	0.06	1.00	1.00
By schooling										
Primary educ.	1.21	0.83	1.22	0.87	0.47	0.47	0.06	0.05	0.23	0.17
Secondary educ. incom.	1.44	1.03	1.34	1.03	0.40	0.41	0.08	0.06	0.16	0.11
Secondary educ. com.	1.66	1.17	1.50	1.17	0.29	0.31	0.08	0.06	0.35	0.35
Tertiary educ. incom.	2.29	1.45	1.88	1.30	0.33	0.34	0.10	0.07	0.09	0.15
Tertiary educ. com.	4.44	3.51	3.56	3.06	0.17	0.21	0.05	0.06	0.17	0.23

Source: Own elaboration using CASEN surveys for waves 2006 and 2017. The wages declared in both years are relative to the minimum wage in the respective year.

During these years, several noteworthy trends emerged in the labor market. As anticipated, wages exhibited a positive correlation with educational attainment, with workers possessing primary education earning, on average, less than the minimum wage in the informal sector. Additionally, it is crucial to highlight that wages in the informal sector also increased in tandem with higher levels of education. This implies that the production technology employed by informal firms benefits from greater human capital, suggesting that, at the very least, they do not appear to be structurally distinct from formal firms in this aspect.

Furthermore, it is worth noting that the educational wage premium exhibited a decline between 2006 and 2017. For instance, in 2006, a formal worker with a complete tertiary education earned 2.67 times the wage of a formal worker with a complete secondary education, whereas by 2017, this premium had reduced to 2.37 times. Furthermore, a notable reduction in the formal wage premium, roughly by 10%, is observed across all educational levels. Several factors contribute to this phenomenon. One of the most prominent factors, as highlighted in the literature, is the

increase in the minimum wage, which elevates the cost of formality and triggers a reallocation of workers for whom the minimum wage becomes a binding constraint.

We should also anticipate a decrease in unemployment. This expectation arises from the increase in the share of high-skilled workers, which should lead to lower unemployment rates. This is because firms, particularly formal ones, are inclined to hire more workers to leverage the augmented marginal productivity resulting from a higher-skilled workforce. In the context of Brazil, this dynamic resulted in a notable 7.4 percentage point reduction in unemployment. However, in the case of Chile, this process did not lead to a substantial decline in unemployment, although it is noteworthy that Chile's initial unemployment rate was significantly lower than that of Brazil (13.1% and 7%, respectively).

4 Baseline model

4.1 Overview

The model presented by Haanwinckel & Soares (2021) is a continuous-time framework that captures various dimensions of labor markets, including search frictions, firm and worker heterogeneity, labor informality, minimum wage regulations, and mandated benefits. Specifically, it focuses on firms' decisions regarding labor informality, primarily exploring the extensive margin of informality. In this model, firms opting for informal employment arrangements benefit from payroll tax savings but face a risk of government detection and subsequent fines. Notably, the model does not incorporate corporate taxes, making the compliance decision with payroll taxes the primary distinction between formal and informal firms.

The model encompasses a population of infinitely lived, income-maximizing workers with linear preferences. Within this population, a fraction η is classified as high-skilled, while the remainder falls into the low-skilled category. Notably, the parameter η is exogenous to the model, as it represents an inherent characteristic of the workforce without involving endogenous decisions regarding educational levels. The model operates within a dual labor market framework at the education level, consisting of separate markets for each skill level. Additionally, there exists a set of risk-neutral profit-maximizing firms denoted by a mass m. These firms engage both high-skilled and low-skilled workers in the production process within the context of a single, economy-wide good.

Firms in this model exhibit heterogeneity on two dimensions. Firstly, they differ in terms of their productivity parameter, denoted as z, which contributes to overall productivity and has the effect of enhancing the marginal productivity of high-skilled workers. This reflects the premise that more productive firms are inclined to disproportionately hire high-skilled workers. Secondly,

firms vary in a penalty parameter, k, which determines the magnitude of fines imposed in the event of discovery. This parameter introduces heterogeneity in the prevalence of informal workers among firms that share the same productivity level. The penalty parameter plays a pivotal role in motivating informal firms to transition into formality, as it incentivizes them to assess the trade-offs associated with being informal or formal, factoring in considerations such as payroll taxes, informal penalties, as well as their unique productivity and penalty parameters.

The equilibrium of this model is characterized by four aggregate variables. The first pair comprises the market tightness values in each labor market, denoted as θ_s and θ_u. These values play a pivotal role as they influence the probabilities of finding a job and filling a vacancy in each respective labor market. The second pair consists of the values associated with unemployment for each skill level, represented as U_s and U_u. These values are instrumental in shaping the bargaining process and, consequently, in determining wage levels.

4.2 Labor markets

In this model, two distinct labor markets are considered, one for each skill level. Consequently, within each market, both formal and informal vacancies coexist simultaneously. This approach aligns with empirical observations discussed in the introduction, as it reflects the rejection of the duality hypothesis by the available data.

In each labor market, there exists a matching function denoted as $M(V_i, u_i)$, where V_i represents the total vacancies available in market i (with $V_i = V_i^{for} + V_i^{inf}$) and u_i denotes the number of unemployed workers of skill level $i \in \{s, u\}$. Notably, there is no differentiation in the visibility of formal and informal vacancies during the search process. Once a match is established, the probability that the offered vacancy is formal is expressed as $\phi_i = \frac{V_i^{for}}{V_i}$. The matching function adheres to standard assumptions, including being increasing in each argument, exhibiting concavity, and possessing constant returns to scale. With these considerations, we can define $q(\theta_i)dt$ as the instantaneous probability of filling a vacancy in market i, and $\theta_i\theta(q_i)$ as the instantaneous probability of finding a job in market i.

The function used to represent the matching function is a Cobb-Douglas. This yields a function of the form $q(\theta) = D\theta^{-E}$ where D is the matching scale and E is the matching elasticity.

Firms operating in each sector $j \in \{s, u\}$ must pay a cost $\Xi_i^j(v)$ for posting vacancies for workers of skill level i. The model assumes that the vacancy marginal cost ($\xi_i^j(v) = \frac{\partial \Xi_i^j(v)}{\partial v}$) is increasing at an increasing rate, following Bertola & Caballero (1994) and Acemoglu & Hawkins (2014). Also

it is assumed that $\Xi_i^j(0) = \xi_i^j(0) = 0$. Then, the proposed vacancy cost function is given by:

$$\Xi_i^j(v) = \xi^j \xi_i \left[v + \frac{\exp(-\xi^S v) - 1}{\xi^S} \right]$$

Where:

$$\xi_i^j(v) = \xi^j \xi_i [1 - \exp(-\xi^S v)]$$

Where the parameters ξ^j and ξ_i allow for different costs of posting vacancies for high and low-skilled workers, informal or formal jobs, respectively. The parameter ξ^S determines the steepness of the function.

These costs can be attributed to various factors, such as the informal firms relying on social networks to find workers, as suggested by Chandrasekhar et al. (2020), or the formal firms incurring costs associated with advertising and utilizing human resources agencies. These assumptions are made with the expectation that firms will not immediately publish the optimal number of job openings they require. Instead, they will gradually accumulate workers over time. Additionally, this setup aims to produce a firm-size premium, a phenomenon observed in data.

4.3 Problem of the firm

There is no entry or exit of firms in this model. Firms have a production function that depend on three parameters: the productivity draw z, the number of high-skilled workers n_s, and the number of low-skilled workers n_u. The functional form used is given by:

$$F^z(n_s, n_u) = Az \left[B(z) n_s^\gamma + (1 - B(z)) \, n_u^\gamma \right]^{\frac{\alpha}{\gamma}}$$

Where:

$$B(z) = \frac{1}{1 + \left(\frac{z}{B_1} \right)^{-B_2}}$$

Where B_1 and B_2 determine the level and slope of the relative weight of high-skilled workers in the production. Given the structure of $B(z)$, it is possible to show that this production function implies that $\frac{\partial^2 F^z(n_s,n_u)}{\partial n_s \partial z} > \frac{\partial^2 F^z(n_s,n_u)}{\partial n_u \partial z}$. The above condition is relevant since it will make that larger firms hire high-skilled workers more than proportional to its productivity levels.

The term z is a productivity parameter that can be interpreted as entrepreneurial talent. This idea was presented in Lucas (1978), and is linked to the assumption that entrepreneurs cannot

efficiently manage a large number of skilled workers if they are not highly talented themselves. It is assumed that the parameter z is distributed across firms with a bounded Pareto distribution with support [1, 10,000]. Then the distribution $G(z)$ is given by:

$$G_z(z) = \frac{1 - z^{-T}}{1 - 10000^{-T}}$$

The parameter T serves to adjust the distribution shape, with a lower T corresponding to a fatter tail in the productivity distribution. The choice of a Pareto distribution stems from its capacity to produce a substantial number of small firms, while the bulk of employment remains concentrated in larger firms, which is reasonable given the Chilean context.

Due to the presence of search frictions, firms are unable to directly hire the workers they require. Instead, they must post job vacancies to initiate the hiring process. Therefore, the model's control variables encompass the number of vacancies posted in each labor market (v_s^j and v_u^j). Additionally, firms need to make a crucial decision regarding compliance with labor regulations. This decision is considered irreversible and must be made at the outset. If a firm chooses to comply, it incurs a payroll tax denoted by τ, which is a percentage of the wages. In contrast, informal firms do not pay the payroll tax but face an informality penalty, represented by $\rho^k(n_s, n_u)$, where $\frac{\partial \rho^k(n_s+n_u)}{\partial n_i} > 0$ and $\frac{\partial^2 \rho^k(n_s+n_u)}{\partial n_i^2} > 0$ for $i \in \{s, u\}$. The specific function used for the penalty is given by:

$$\rho^k(n_s + n_u) = p_k \cdot (n_s + n_u)^{P^E}$$

Where:

$$p_k = \exp(P + (k - 3)P^D)$$

And k is a firm-specific enforcement parameter with discrete uniform distribution with support $\{1, 2, 3, 4, 5\}$. P is a measure of enforcement of regulation (corresponding to the median of the enforcement intensity) and P^D is a measure of dispersion. The heterogeneity in k can be seen as structural differences across sectors that make it harder for some firms to hide of labor regulation. Also z and k are assumed to be independent.

It is crucial that the penalty depends on the total number of workers within a firm, as this accounts for the notion that informality costs escalate with firm size.[4] Consequently, larger firms will experience relatively greater advantages from maintaining formal status compared to smaller firms.

[4]The penalty can be interpreted as the anticipated cost of fines incurred when discovered by labor inspectors. This interpretation aligns with explanations provided in other studies (Meghir et al., 2015; Cisneros-Acevedo & Ruggieri, 2022; Parente, 2022)

Then, after normalize the price of the final good to 1, the instantaneous profit function with productivity draw z, penalty parameter k and compliance decision j is given by:

$$\psi^{z,k,j}\left(n_s,n_u,v_s,v_u\right)=\begin{cases} F^z\left(n_s,n_u\right)-\sum\limits_{i=s,u}\left[(1+\tau)n_i w_i^{z,for}\left(n_s,n_u\right)+\Xi_i^{for}\left(v_i\right)\right], \text{if } j=\text{for} \\ F^z\left(n_s,n_u\right)-\rho^k\left(n_s+n_u\right)-\sum\limits_{i=s,u}\left[n_i w_i^{z,k,inf}\left(n_s,n_u\right)+\Xi_i^{inf}\left(v_i\right)\right], \text{if } j=\text{inf} \end{cases}$$

Where $w_i^{z,for}\left(n_s,n_u\right)$ and $w_i^{z,k,inf}\left(n_s,n_u\right)$ are wages that the firm (z,k) pays to workers of type i, according to its compliance status j and the current number of employees, n_s and n_u. The process to determine wages is presented in the next subsection. Then, instantaneous profits are given by the total production minus the payroll payments, vacancy-posting costs, and either payroll taxes or the informality penalty according to the compliance decision.

Concerning job relations, they exhibit an exogenous destruction rate denoted as λ_i^j, contingent upon both skill level and the sector of the vacancy. This approach aims to account for differences in job durations between low-skilled and high-skilled workers, as well as between formal and informal positions. Specifically, it enables the model to emulate the higher turnover commonly observed among low-skilled workers and in informal employment. Consequently, the dynamics of labor within each firm can be expressed as follows:

$$\dot{n}_i = v_i q(\theta_i) - \lambda_i^j n_i$$

Where $i \in \{s,u\}$ and $j \in \{\text{for,inf}\}$. From this equation we can understand that the movement in the number of workers employed in firm depends on the number of vacancies filled minus the number of job destructions.[5]

Therefore the problem of the firm is given by:

$$\begin{aligned} \Pi^{z,k} &= \max\nolimits_{j\in\{for,inf\}}\Pi^{z,k,j}\left(n_s^{z,k,j,\text{initial}},n_u^{z,k,j,\text{initial}}\right), \text{with} \\ \Pi^{z,k,j}\left(n_s,n_u\right) &= \max\nolimits_{\{v_s,v_u\}}\left(\tfrac{1}{1+rdt}\right)\left\{\psi^{z,k,j}\left(n_s,n_u,v_s,v_u\right)dt+\Pi^{z,k,j}\left(n_s^+,n_u^+\right)\right\} \\ \text{s.t. } n_i^+ &= n_i+\dot{n}_i dt=\left(1-\lambda_i^j dt\right)n_i(t)+v_i q\left(\theta_i\right)dt, \quad i=\text{s,u} \end{aligned} \tag{1}$$

For a firm with productivity z and enforcement intensity k, given a compliance decision j, the total present value of profits is the sum of instantaneous profits earned at the end of the short time interval dt plus the present value of profits after dt. The discount rate r is the same for all

[5]This equation implies that every match turns into in a job relation. This is because it is assumed that after every match there is a Nash bargaining process over the rents created by the match. Given this every agent will receive more than its reservation value, and therefore, every match will turn into a job relation.

firms. Given its initial conditions and productivity, the firm makes the compliance choice that maximizes the present discounted value of profits. The model restricts attention to steady-state solutions where the numbers of workers of different types are constant in each firm.[6]

With the problem stated, it is possible to continue and obtain the first order conditions. Given the control variables, we take two partial derivatives, one for vacancies and another for employment. Denoting $\pi_i^{z,k,j}(n_s, n_u)$ the marginal value of an additional worker of type i in a firm of type z, k with compliance decision j, i.e., $\pi_i^{z,k,j}(n_s, n_u) = \frac{\partial \Pi_i^{z,k,j}(n_s,n_u)}{\partial n_i}$. After imposing the steady-state condition, it is possible to show that the FOCs are:

$$\left(r + \lambda_i^j\right) \pi_i^{z,k,j}\left(n_s, n_u\right) = \begin{cases} F_i^z\left(n_s, n_u\right) - (1+\tau)\left[w_i^{z,for}\left(n_s, n_u\right) + \sum\limits_{l=s,u} n_l \frac{\partial w_l^{z,for}(\cdot)}{\partial n_i}\right], \text{for } j = \text{for} \\ F_i^z\left(n_s, n_u\right) - w_i^{z,k,inf}\left(n_s, n_u\right) - \sum\limits_{l=s,u} n_l \frac{\partial w_l^{z,k,inf}(\cdot)}{\partial n_i} - \rho^{k\prime}\left(n_s + n_u\right), \text{for } j = \inf \end{cases} \tag{2}$$

And:

$$\pi_i^{z,k,j}\left(n_s, n_u\right) = \frac{\xi_i^j\left(v_i\right)}{q\left(\theta_i\right)} \tag{3}$$

Where $F_i^z\left(n_s, n_u\right) = \frac{\partial F^z(n_s,n_u)}{\partial n_i}$ and $\rho^{k\prime}\left(n_s + n_u\right) = \frac{\partial \rho^k(n_s+n_u)}{\partial n_i}$.

Equation (2) shows the marginal value of a worker. It is composed of a classical term which is the increase in the marginal productivity minus the cost of the wage, and of a new term which is the change in the payroll given the change in all marginal productivities. At the time of the hiring decision or bargaining, previous vacancy costs are sunk and thus do not appear in this expression.

Equation (3) is the optimality condition in a steady state and says that the value of the marginal worker must be equal to the expected cost of hiring another worker. By combining both expressions, they find the first-order condition of the formal firm analogous to the one in Cahuc et al. (2008), in which marginal product equals a generalized notion of marginal cost:

$$\underbrace{F_i^z\left(n_s^{z,\text{ for}}, n_u^{z,\text{ for}}\right)}_{\substack{\text{Marginal} \\ \text{productivity}}} = (1+\tau)\underbrace{w_i^{z,\text{ for}}\left(n_s^{z,\text{ for}}, n_u^{z,\text{for}}\right)}_{\substack{\text{Own} \\ \text{wage}}} + (1+\tau)\underbrace{\sum_{l=s,u} n_l^{z,\text{ for}} \frac{\partial w_l^{z,\text{ for}}(\cdot)}{\partial n_i}}_{\substack{\text{Effect on other} \\ \text{Workers' wages}}} + \underbrace{\frac{r + \lambda_i^{\text{for}}}{q\left(\theta_i\right)} \xi_i^{\text{for}}\left(\frac{\lambda_i^{\text{for}}\, n_i^{\text{for}}}{q\left(\theta_i\right)}\right)}_{\substack{\text{Hiring} \\ \text{costs}}}$$

Where they denote the optimal labour choices in firm z as $n_s^{z,for}$ and $n_u^{z,for}$ (as opposed to arbitrary choices n_s and n_u).

[6] Given that we are studying the steady-state solutions, it could be worth asking why we still assume the convex costs for posting vacancies. The above is that because of them, we can generate size premiums.

4.4 Wage determination

Wages are determined through Nash bargaining, with workers and firms sharing the rents created by the match. The share of the surplus appropriated by a worker is given by the exogenous parameter σ, which corresponds to workers' bargaining power, and workers and firms engage in renegotiation after the initial match.[7] Firms must consider that changes in their size will result in wage renegotiation, as hirings will change the marginal productivities (Stole & Zwiebel, 1996). To solve this, an extension of the solution proposed by Cahuc et al. (2008) is used.[8]

Let $J_j^i(w)$ represent the value of a job for workers of type i in sector j, where the wage offered is w. Additionally, let U_i denote the value of unemployment for a worker of type i.Then the value of a job in the formal and informal sector is given by:

$$rJ_i^{for}(w) = a_i w + b_i + \lambda_i^{for}\left[U_i - J_i^{for}(w)\right] \tag{4}$$

$$rJ_i^{inf}(w) = w + \lambda_i^{inf}\left[U_i - J_i^{inf}(w)\right] \tag{5}$$

where a_i and b_i represent mandated benefits that may increase (or decrease) the value of holding a formal job.[9]

Considering that the marginal benefit of the firm is given by $\pi_i^{z,k,j}(n_s, n_u)$ and the benefit of a worker of accepting a job is equal to $rJ_i^j(w)$, then without considering the minimum wage the Nash bargaining sharing rule implies that the wage $w_i^{z,k,j}(n_s, n_u)$ must satisfy:

$$(1-\sigma)\left[J_i^j\left(\, w_i^{z,k,j}(n_s, n_u)\,\right) - U_i\right] = \sigma\pi_i^{z,k,j}(n_s, n_u) \tag{6}$$

This is a system of nonlinear differential equations since the component $\pi_i^{z,k,j}(n_s, n_u)$ includes the

[7] It is noteworthy that Haanwinckel & Soares (2021) have opted not to distinguish between workers' bargaining power in the informal sector. Unlike some prior studies, such as Ulyssea (2010) and Bosch & Esteban-Pretel (2012), which have posited varying bargaining powers, the authors defend their stance by contending that divergent bargaining powers can directly lead to different labor market outcomes, potentially obscuring the underlying factors driving disparities between the formal and informal sectors. Consequently, by assuming equal bargaining power in both sectors, any variations in labor market outcomes can be attributed solely to the structural characteristics of the model.

[8] Cahuc et al. (2008) solves the problem for one worker type, while Haanwinckel & Soares (2021) extend the solution to two worker types.

[9] In a search frictions framework, it's crucial to consider that U_i can encompass unemployment benefits, which typically depend on eligibility criteria specific to each unemployment system. This aspect is simplifying by assuming that agents are risk-neutral and receive the corresponding discounted value of unemployment benefits while they are employed. It's worth noting that while this assumption simplifies the model, it's not devoid of generality, as variations in unemployment benefit values could influence the search decisions of unemployed agents, as seen in the literature (Pissarides, 2000; Fredriksson & Holmlund, 2001, 2006).

derivative of wages.The solution is given by:

$$w_i^{z,for}(n_s,n_u) = \frac{1-\sigma}{c_i}(rU_i - b_i) + \frac{1}{1+\tau}\int_0^1 \epsilon^{\frac{1-\sigma}{\sigma}\frac{a_i}{1+\tau}} \frac{\partial F^z\left(\epsilon^{\frac{a_i}{a_s}}n_s, \epsilon^{\frac{a_i}{a_u}}n_u\right)}{\partial n_i} d\epsilon, \text{ and}$$

$$w_i^{z,k,inf}(n_s,n_u) = (1-\sigma)rU_i + \int_0^1 \epsilon^{\frac{1-\sigma}{\sigma}} \frac{\partial H^{z,k}(\epsilon n_s, \epsilon n_u)}{\partial n_i} d\epsilon$$

Where $c_i = [(1-\sigma)a_i + \sigma(1+\tau)]$ and $H^{z,k}(n_s,n_u) = F^z(n_s,n_u) - \rho^k(n_s+n_u)$.

Regarding the minimum wage, the former will be binding if the bargained wage in a formal firm for one type of worker (usually low-skilled workers) is lower than the minimum wage. In that case the Nash bargaining equation is not valid any more as the low-skilled workers will have a share higher than σ. The above will affect also the bargaining process of the high-skilled workers, since the term $\frac{\partial w_u^{z,for}}{\partial n_s}$ becomes zero. This is because marginal changes in the number of high-skilled workers will not change the low-skilled wages. Then, a new solution is needed, and is given by:

$$w_i^{z,for}(n_s,n_u) = \frac{1-\sigma}{c_s}(rU_s - b_s) + \frac{1}{1+\tau}\int_0^1 \epsilon^{\frac{1-\sigma}{\sigma}\frac{a_s}{1+\tau}} \frac{\partial F^z(\epsilon n_s, n_u)}{\partial n_i} d\epsilon$$

Then, the minimum wage introduces a discontinuity in the first-order condition of the firm. Without a minimum wage and considering the complementarities across types, hiring an additional skilled worker decreases high-skilled wages and increases low-skill wages. The reverse is true for hiring an low-skilled worker. This effect is considered into account in the value of the marginal worker of both types. However, when the minimum wage becomes binding for unskilled workers, the effect of firm size on unskilled wages disappears, leading to a discontinuous increase in the marginal benefit of high-skilled workers ($\pi_s^{z,k,for}$) and the opposite for the marginal benefit of low-skilled workers ($\pi_u^{z,k,for}$).

The above is interesting because it can generate strategic behavior in the firm. The firm can reduce the number of unskilled workers or increase the number of skilled workers just enough so that bargained unskilled wages are slightly above the minimum wage. More interesting is the possibility that when the unconstrained (freely bargained) unskilled wage is slightly lower than the minimum wage, the firm problem might not be solved. In that case it is assumed that the firm will pay the highly skilled workers the wage associated with the payment of the minimum wage to the low-skilled workers, considering that the latter was freely agreed upon.

4.5 Equilibrium

The equilibrium determines the value of the endogenous variables in the model, the market tightness in each market (θ_s, θ_u) and the value of unemployment for each worker's type (U_s, U_u).

To compute those, first we need to define the total employment of workers type i in the sector j. If it is defined by N_i^j, then it is given by:

$$N_i^j = m \int \int n_i^{z,k,j} \mathbb{1}(\text{Firm } z,k \text{ chooses compliance } j) dG_z(z) dG_k(k)$$

Since in equilibrium there is no movement in the number of workers in each firm ($\dot{n}_i = 0$), the vacancies posted will be given by $v_i^{z,k,j} = \lambda_i^j n_i^{z,k,j}/q(\theta_i)$. Then, the total amount of vacancies in each market will be $V_i^j = \lambda_i^j N_i^j / q(\theta_i)$. With the above it is possible to pin down expressions for the market tightnesses:

$$\theta_s = \frac{\lambda_s^{for} N_s^{for} + \lambda_s^{inf} N_s^{inf}}{q(\theta_s)(\eta - N_s^{for} - N_s^{inf})} \quad \text{and} \quad \frac{\lambda_u^{for} N_u^{for} + \lambda_u^{inf} N_u^{inf}}{q(\theta_s)(1 - \eta - N_u^{for} - N_u^{inf})} \tag{7}$$

Finally, the value of unemployment is given by the weighted average (given by the probabilities of each case) of being unemployed (and receive d_i), being employed in the formal sector and receive the expected utility of that job ($a_i E[w_i^{for}] + b_i$), and being employed in the informal sector and receive the expected utility of that job ($E[w_i^{inf}]$):

$$rU_i = \frac{1}{\frac{1}{\theta_i q(\theta_i)} + \frac{\phi_i}{r+\lambda_i^{for}} + \frac{(1-\phi)}{r+\lambda_i^{inf}}} \left[\frac{1}{\theta_i q(\theta_i)} d_i + \frac{\phi_i}{r + \lambda_i^{for}} \left(a_i E[w_i^{for}] + b_i \right) + \frac{(1-\phi)}{r+\lambda_i^{inf}} E[w_i^{inf}] \right] \tag{8}$$

Where $\phi = \frac{V_i^{for}}{V_i^{for} + V_i^{inf}} = \frac{\lambda_i^{for} N_i^{for}}{\lambda_i^{for} N_i^{for} + \lambda_i^{inf} N_i^{inf}}$ is the probability of having a match with a formal firm given the probability of having a match.

An equilibrium in this model is defined as set of wage functions $w_i^{z,k,j}(n_s, n_u)$, schedules of firms decisions $j(z)$ and $n_i^{z,k,j}$, labor market tightnesses θ_i, and unemployment values U_i, such that:

1. The wage functions solve the system of differential equations given by equations 2 and 6.
2. The labour schedules $n_i^{z,k,j}$ solve equation 3 given the compliance decision $j(z)$ and the wage functions
3. The compliance decisions $j(z)$ maximize the present value of discounted profits in problem 1
4. The labour market tightnesses are consistent with equation 7
5. The unemployment values are consistent with equation 8.

5 Estimation

5.1 Data Reduction: from Age-Education to Skill

This section will present the estimation procedure of the baseline model. The first step is to define what a skilled worker is. Most of the literature that uses two types of workers and present a search model with minimum wage uses the fact of holding a college degree as indicator of being a skilled worker (Ulyssea, 2018; Dix-Carneiro et al., 2021; Vergara, 2022). Instead of doing this, Haanwinckel & Soares (2021) use an endogenous procedure in which data will determine the amount of skilled workers.

First, we need to divide all the workers in 12 age-education groups. The age groups will be given by the ages [18-29, 30-39, 40-49, 50-65], while the education groups will be given by the years of schooling between [0-11, 12, 13+].[10] Then we need to assume the following:

1. There are two latent worker types, skilled and unskilled; each one of the twelve age–education groups is a convex combination of skilled and unskilled workers.
2. The first group (18-29 years old, less than 12 years of schooling) is composed of unskilled workers only, and the last group (50-65 years old, 13 or more years of schooling) is composed of skilled workers only.
3. There are no job-to-job transitions.

Then we need to solve an optimization problem where choice variables are skill shares in each age–education group (except the first and the last), job destruction parameters, job-finding rates by skill, and a series of skill-level variables corresponding to various labor market moments. The goal of this optimization problem is to minimize deviations between the age–education moments and the corresponding moments implied by the region–skill-level summary.

The loss function of this problem is given by:

$$\mathcal{L} = \sum_{l=1}^{11} \sum_{g=1}^{12} S_g \left[M_{g,l}^{Data} - \left((1-\omega_g) M_{u,l}^{Model} + \omega_g M_{s,l}^{Model} \right) \right]^2$$

Where ω_g represent the share of skilled workers in each group g.

To compute this, I use two datasets. First, I use the national employment survey (ENE) to compute the moments associated with the labor transitions and the CASEN survey to compute

[10] This ranges are different from those used in the baseline paper, I took this ranges because they adequate better to the Chilean context. In the Appendix there is the detail about the changes.

the rest of the moments. The above is because CASEN is a cross-sectional survey and does not allow computing transitions. In contrast, the ENE survey allows for estimating labor transitions but does not have information about wages.

After doing this procedure, the matrix of skill composition is presented in Table 2. As can be seen, the shares of skilled workers follow the logic that one could expect. Given any age group, an increase of the education level leads to a higher share of skilled workers. The same happens for any educational group and an increase of the age.

Table 2: Shares of skilled workers in each Age-Education group

	Education groups		
Age groups	0-11	12	13+
18-29	0.00	0.16	0.53
30-39	0.03	0.27	0.79
40-49	0.04	0.33	0.84
50-65	0.03	0.40	1.00

With this, I can obtain the share of skilled (η) and unskilled ($1-\eta$) workers in the country, and also the implied parameters associated with the destruction rates of jobs for each skill-sector combination ($\lambda_s^{for}, \lambda_s^{inf}.\lambda_s^{inf}$ and λ_u^{inf}).

5.2 Imputed parameters

Given the procedure explained in the subsection below, it is possible to obtain five parameters that are going to be imputed to the model. However, there are another group of parameters that need to be imputed.First, we need a measure of firms, that is, a parameter that allows to compare the ratio between firms and workers. To compute this parameter I use public data from the tax office in Chile. Second, it is necessary to impute the values associated with the payroll taxes, mandated benefits, and unemployment benefits. I follow the same procedure used by Haanwinckel & Soares (2021) to impute those parameters.[11]

Third, we need some parameters to govern the matching function. In this case, and as the authors, I will assume the values commonly used in the literature. It is worth to mention that I will use same estimation of the elasticity of substitution between skilled and unskilled workers (a key parameter given the underlying idea of the model) than the authors, which comes from

[11]The primary difference is related to fixed benefits for workers. In Chile, these parameters are set at 0 for both skilled levels, whereas in Brazil, the values are 0.02 and 0.05, respectively.This difference is due, in Brazil, workers who achieve a certain tenure are granted a bonus from the government which is a fraction of the minimum wage. This is not the case in Chile.

Fernández & Messina (2018).[12].

The summary of these parameters can be seen in Table 3.[13] As it can be seen, the parameters estimated through the reduction data process represent facts that can be reasonable to sustain. The job duration in skilled job is larger than the duration in unskilled jobs, and the duration of formal jobs is larger than the duration of informal jobs.

Table 3: Imputed parameters

Parameter	Value	Source
Panel A: Estimated from data reduction		
η (share of skilled workers at the nationall level)	0.2188	Estimated from CASEN and ENE
λ_s^{for} (formal skilled job destruction)	0.0131	Estimated from CASEN and ENE
λ_s^{inf} (informal skilled job destruction)	0.0289	Estimated from CASEN and ENE
λ_u^{for} (formal unskilled job destruction)	0.0149	Estimated from CASEN and ENE
λ_u^{inf} (informal unskilled job destruction)	0.0329	Estimated from CASEN and ENE
m (measure of firms)	0.0790	Estimated from SII data and CASEN
Panel B: Formal labor parameters		
τ (payroll tax)	0.6507	Own elaboration
a_s, a_u (variable benefits)	[0.293; 0.495]	Own elaboration
b_s^F, b_u^F (fixed benefits)	[0.000; 0.000]	Own elaboration
b_s^D, b_u^D (unemployment insurance)	[6.556; 3.761]	Own elaboration
Panel C: Matching function parameters and discount factor		
r (discount rate)	0.008	Haanwinckel & Soares (2021)
σ (bargaining power of workers)	0.5	Assumed symmetric
D (matching scale)	0.3	Ulyssea (2010)
E (matching elasticity)	0.5	Ulyssea (2010)
γ (elasticity of substitution = $\frac{1}{1-\gamma}$)	0.436	Fernández & Messina (2018)

5.3 Estimation procedure

The estimation procedure has three steps. First, we need to gather the moments associated with the labor market from a micro perspective (such as average wages). This data will be obtained from CASEN and ENE surveys. The second step is to obtain some moments associated with the labor market from a macro perspective (such as the share of labor income). These moments will be obtained through national account information and the administrative records of the unemployment insurance system (SC). Finally, I will estimate the parameters in the third step using a minimizing distance procedure.

CASEN and ENE moments. The moments used are the following: informality rates,[14]

[12]In this case, it is reasonable use that parameter since it is estimated using appended data of Argentina, Brazil and Chile.

[13]In the Appendix can be found a comparison between the imputed values for both Brazilian and Chilean cases.

[14]This uses the definition created in the Section 2.

mean log hourly wages, formal wage premium, firm-size wage premiums (for firms with 6–10 and 11 or more workers, relative to the excluded category with 1–5 workers),[15] job finding rates (transitions from unemployment to any kind of employment), and distribution of workers across firm sizes in the formal and informal sectors (shares of formal and informal workers in firms with 6–10 employees, and 11 or more employees). Most of them are calculated by taking averages of variables at the individual level (with survey weights) and using the composition of skills by age-education groups discussed in the data reduction procedure.

National accounts and administrative records moments. The model uses three additional moments from a macro perspective. The first one is the labour share in national income. The labour share is helpful for identifying the returns to scale parameter of the production function, α. In the model, labour share is defined as the fraction of total production (net of search costs and informality penalties) that is not firm profits nor government surplus. The second moment is the effect of enforcing labor regulations on the informality rate, where we used the the elasticity computed by Haanwinckel & Soares (2021).[16]

The third national-level target is the ratio of workers employed in formal firms with more than 500 employees to those in formal firms with more than 100 employees. This moment is relevant to estimate the distribution parameter T. I used a sample of the administrative records associated with unemployment insurance (SC) to obtain this parameter. The sample is a 3% random selection of registered employers and shows information about all the employees working in those firms. Given the above, I can compute the ratio between the employment levels in the two thresholds. Of course, the SC database only shows data on formally employed workers in formal firms. However, this should not be a problem since it is reasonable to expect the number of informal firms with more than 100 workers to be minimal and even more for informal firms with 500 workers.

Minimum distance estimation. The remaining parameters of the model are estimated by minimizing the distance between the 24 moments collected in previous steps and their predicted values their corresponding values implied by the model. It is possible to define a function $h : X \to \mathbb{R}^{24}$ which takes a set of parameters $x \in X \subset \mathbb{R}^{14}$, finds the equilibrium of the model, and simulates the corresponding moments by integrating over the distribution of firms.

[15]The wage premiums are obtained from panel regressions run separately for each region-age-education group. The dependent variable is the log hourly wage and the controls are individual fixed effects, and a linear term in age. For the formal wage premium, the main independent variable is formality status. Firm-size wage premiums are obtained from similar regressions that control for firm size categories. Wage premiums are aggregated from age-education groups to skills using the shares obtained from the data reduction procedure.

[16]The authors follow Almeida & Carneiro (2012), who show that enforcement is strongly predicted by the interaction of two variables: the number of labor inspectors based in the closest office of the Ministry of Labor and the travel distance (in hours) between that office and the municipality. Then Haanwinckel & Soares (2021) run an instrumental variables regression to find the effect of enforcement on the municipality-level informality rate, using the interaction as the instrument and the same set of controls that Almeida & Carneiro (2012) used.

Then the estimation procedure finds the parameters x that minimize a weighted sum of squared relative errors, using a gradient descent algorithm and parallel processing to analyze a large number of starting points. Weights are proportional to the share of the workforce from which each moment was calculated. Finally, the algorithm requires some initial values to start the iteration to find the parameters that minimize the loss function. Since the values obtained once the process has finished depend highly on the initial points, for each seed used we generate five sets of initial points. Then the algorithm chooses the one that minimizes the loss function and starts the iteration. This process involves the use of 200 seeds, which gives around 1,000 sets of initial points.

5.4 Estimated parameters

Estimated parameters are shown in Table 4. The flow utility of unemployment is small for unskilled workers and strongly negative for skilled workers. The above goes in line with Hornstein et al. (2011) who show that small or negative leisure values are necessary to match realistic levels of wage dispersion for similar workers in matching models.

Table 4: Estimated parameters

Parameter	Value
Production function	
A (productivity shifter at the national level)	6.5909
B_1 (skill bias shifter)	1.4985
B_2 (relative skill bias in productive firms)	0.1974
α (decreasing returns)	0.4326
Distribution of firms	
T (tail parameter)	4.5882
Informality cost	
P (enforcement shifter)	-6.3221
P^D (enforcement penalty dispersion)	11.8162
P^E (firm size exponent)	4.5882
Vacancy-posting cost	
ξ^{for} (formal sector component)	91.1363
ξ^{inf} (informal sector component)	0.81504
ξ_s (skilled worker shifter)	73.2099
ξ^S (steepness)	0.0049
Utility flow while unemployed	
d_s (skilled workers)	-6.5862
d_u (unskilled workers)	-6.6943

As in Haanwinckel & Soares (2021), vacancy-posting costs are high in the skilled labour market, but negligible for informal firms. This implies that match rents are small on the informal sector, and therefore informal workers are close to be indifferent between unemployment. Regarding the

rest of the parameters, it is not trivial to give them a straightforward interpretation, so I will comment the aggregate effects over the equilibrium.

5.5 Model fit and validation

However, before that, it is worth seeing the fit of the model and the validation process, both presented in Table 5. Regarding the model fit, it is possible to see that it generally has a good adjustment to the data, although some aspects are difficult for the model to explain. First, we can see that the prediction for the informality rate is correctly measured by the model, with an slight difference in the informality level for skilled workers. For unemployment, we can see a good model fit for the unskilled workers, while there are some differences for the skill ones. Regarding wages, there is a good fit for skilled workers, although there are significant differences for unskilled workers.

The main differences between data and model predictions are given in the formal wage premium and the shares of workers in firms with between 6 and 10 employees. These two facts are related to the model structure.[17] Finally, the model presents a good fit regarding the labor share of income, the employment ratio between firms with more than 500 and more than 100 workers, and the enforcement elasticity of informality.

In addition, Figure 4 presents the cross-sectional characteristics of the model in equilibrium. In the graphs related to wages, we observe compensating differentials in the wages of skilled workers, as informal jobs offer higher wages than formal jobs in firms with similar productivity levels. This phenomenon does not apply to unskilled workers due to the binding minimum wage, which effectively increases their bargaining power and raises wages for these workers in the formal sector. Furthermore, the figure illustrates that the minimum wage is binding for formal firms until they reach medium productivity levels.

It is possible to see that employment increases with the productivity level and that informal firms' employment is higher for informal firms facing lower informality parameters. However, this does not imply that the proportion of skilled workers increases with the firm's size. As we can see, it grows initially but then decreases until the distribution's right tail rises again. This occurs because the increase in hiring costs, given by the convex costs, is more significant than the increase in productivity. Therefore firms fail to maintain the proportion of high-skilled workers increasing. Finally, firm and employment distributions show that informal firms are restricted to low productivity levels.

[17] Since the model cannot generate sufficiently large firm size premiums for firms in the middle of the distribution, this will affect the wage premiums of that point, affecting wages and the share of workers across different firm sizes.As Haanwinckel & Soares (2021) mentions, this probably occurs due to parametric restrictions regarding z, k, and vacancy posting costs.

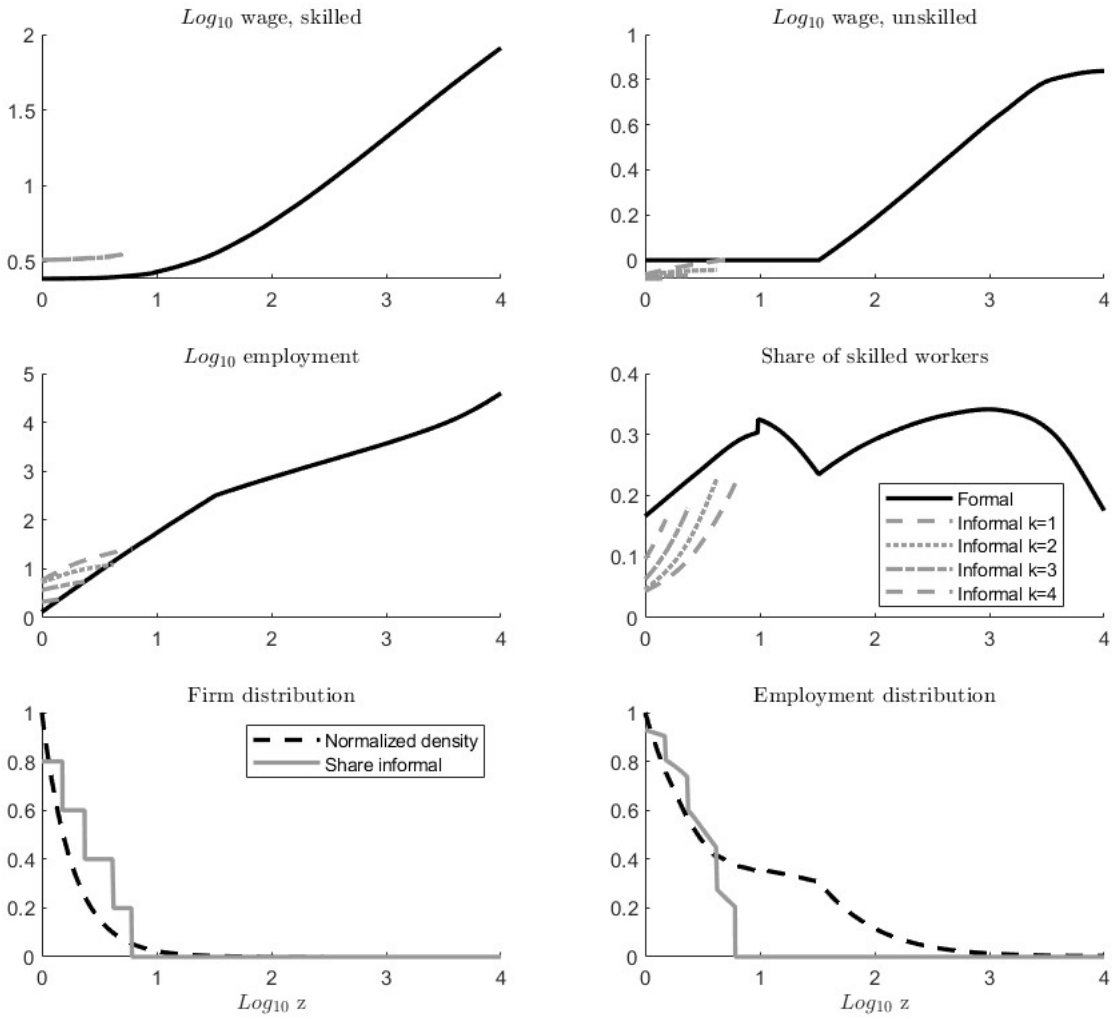
Table 5: Model fit and validation 2006-2017

	Fit in 2006		Validation: Δ 2006-2017	
Outcomes	Data 2006	Model 2006	$\Delta\text{Data}_{2006-2017}$	$\Delta\text{Model}_{2006-2017}$
Informality rate	0.3610	0.3676	-0.0313	-0.0524
Skilled	0.1600	0.1705	0.0536	0.0087
Unskilled	0.4173	0.4228	-0.0173	-0.0252
Unemployment	0.0552	0.0638	-0.0013	0.0291
Skilled	0.0695	0.1051	0.0041	0.0315
Unskilled	0.0512	0.0522	-0.0093	0.0142
Wages (ln)	0.2396	0.2846	0.5809	0.4318
Skilled	1.2270	1.2346	0.2288	0.1667
Unskilled	-0.0372	0.0182	0.4724	0.2826
Formal wage premium	0.3297	0.2014	-0.1219	0.0519
Skilled	0.4391	0.0399	-0.1925	0.0225
Unskilled	0.2991	0.2466	-0.1148	0.1224
Firm-size premium 6-10 workers	0.0811	0.0115	-0.0233	-0.0076
Skilled	0.0215	0.0211	0.0603	-0.0324
Unskilled	0.0978	0.0088	-0.0545	0.0043
Firm-size premium 11+ workers	0.2985	0.2353	-0.0553	0.0544
Skilled	0.4501	0.2848	-0.0463	-0.0111
Unskilled	0.2559	0.2214	-0.1102	0.0779
% formal workers firms 6-10 employees	0.1272	0.0714	-0.0354	-0.0220
Skilled	0.0714	0.0610	-0.0088	-0.0020
Unskilled	0.1429	0.0743	-0.0332	-0.0307
% formal workers firms 11+ employees	0.7786	0.8506	0.0605	0.0465
Skilled	0.8987	0.8834	0.0162	-0.0006
Unskilled	0.7449	0.8415	0.0482	0.0644
% informal workers firms 6-10 employees	0.2913	0.4255	-0.0531	-0.1084
Skilled	0.2632	0.3198	-0.0485	-0.0109
Unskilled	0.2992	0.4551	-0.0467	-0.1331
% informal workers firms 11+ employees	0.2821	0.3572	0.0413	-0.1160
Skilled	0.3559	0.4458	0.0660	-0.1602
Unskilled	0.2614	0.3324	0.0022	-0.1180
Labor share	0.3850	0.3716	0.0500	0.0389
Enforcement elasticity of informality	-0.0534	-0.0535		
Ratio E_{500}/E_{100}	0.3350	0.3426		

The validation exercise involves estimating predicted values for the year 2017 and comparing them with the actual data for that year. To do this, certain model parameters need to be updated to reflect the changing labor market conditions in 2017. Table 6 provides a summary of these updated parameters.

One of the updates is related to the minimum wage. In order to account for changes in the real minimum wage between 2006 and 2017, we calculate that the minimum wage in 2017 is 1.36 times the real minimum wage in 2006, indicating a 36% increase in real terms over that period.

Figure 4: Cross-sectional features of the equilibrium



Notes: The top four panels show firm choices in equilibrium for different values of firm productivity z and informality cost shifter k. Conditional on being formal, k is irrelevant, and thus there is a single curve for formal firms in each panel. Different curves for informal firms reflect different values of k. These curves do not cover the whole range of productivities because, when z is high enough, firms choose to comply with regulations. Because firms with k=4 and k=5 are formal for all values of z, there are only three such curves. The bottom two panels show distributions of firms and total employment by firm productivity z. They also show shares of firms of a given z, or shares of workers employed in those firms, that are informal.

The next step involves updating the share of skilled workers in the labor force. To do this, we calculate the share of workers in each age-education group in 2017 and then use Table 2 from the data reduction procedure to compute the new share. The updated share of skilled workers is determined to be 0.38, which represents a 72% increase between 2006 and 2017. This substantial increase can be attributed to two main factors: firstly, the previously mentioned surge in tertiary education within the country, and secondly, the aging of the population. The model suggests that

workers with the same level of education but who are older have a higher probability of being classified as high-skilled. It is worth noting that the increase in the minimum wage is approximately half of the increase in the share of skilled workers during this period.

Subsequently, utilizing the updated proportion of skilled workers, we can compute the revised job destruction rates for each skill and sector combination. Interestingly, we observe an increase in destruction rates for skilled workers in the formal sector, while they remain constant for unskilled workers from 2006 to 2017. However, during the same period, informal job destruction rates for both worker types decrease, with a particularly significant reduction for unskilled workers. This implies that skilled workers have less value on the formal sector, prompting a shift toward hiring unskilled workers. The overall impact of these changes will depend on how the relative prices influence hiring decisions.

Up to this point, the replication exercise closely follows the methodology outlined in Haanwinckel & Soares (2021). However, at this juncture, I will introduce some modifications. So far, we have applied changes only considering the parameters associated to the labor market, but other parameters are changing between 2006-2017. The baseline model adjusts the parameters A (TFP) and B_2 (relative skill bias in productive firms) to reflect the fact that the production process is changing. I will also update those parameters, but I will include other changes that are relevant, namely I will include: payroll taxes, variable benefits, measure of firms, enforcement shifter, decreasing returns and the skill bias shifter.

First, regarding the productivity, between 2006 and 2017 the TFP fell around 3.4% (CNEP, 2017). Therefore, I will adjust the parameter A downwards in that proportion. About payroll taxes and variable benefits, these parameters must be updated due to a significant change in the AFP system commission during this period. In 2010, a new process was introduced in which new affiliates to the system are put out to tender, and the AFP with the lowest commission takes them all. This led to substantial changes in commission structures, reducing the system-wide average from 2.40% in 2006 to 1.14% in 2017 (SP, 2006, 2017). This change has notable implications for payroll taxes, resulting in a decrease in this parameter when incorporated. The same applies to associated benefits, as having a formal job has become less costly.

Another parameter that requires adjustment is the measure of firms. Between 2006 and 2017, several policies were implemented to increase the number of firms in the country. By utilizing statistics from the Chilean tax bureau (SII, 2023), we can calculate that the ratio of firms to employees increased from 0.079 to 0.087 during the study period, and we incorporate this change into the model. A higher measure of firms implies greater competition for workers, higher wages, and lower unemployment, although it may also lead to higher informality rates.

Finally, the last adjustment concerns the enforcement shifter parameter. In the baseline paper, the authors updated this parameter by increasing it in the same proportion as the increase in the number of processed worker claims. Based on data from the Chilean labor office (DT, 2017), between 2006 and 2017, the number of processed worker claims increased by 20.9%. Therefore, this parameter also faces the same increase.

Taking all of this into consideration, the final step in the estimation process is to update the parameters associated with decreasing returns (α), the skill bias shifter (B_1), and the relative skill bias in productive firms (B_2). I will adjust the decreasing returns to scale parameter because during that period, Chile experienced a significant increase in the labor share. In 2006, the labor share in the country was approximately 0.38, while in 2017, it was around 0.44. This increase is substantial, and the model cannot account for it solely through adjustments in the bias parameters. With this in mind, I will choose the changes in these three parameters that minimize the differences in wages and unemployment for skilled and unskilled workers.[18].

In light of the observations, the model estimates that the parameter α increases by 12%, while B_1 decreases by 83%, and B_2 decreases by 2.5%. The increase in α can be interpreted as a general increase in wages for all workers, while the decrease in B_1 suggests that all firms in the economy are heavily increasing their demand for skilled workers. Conversely, the decrease in B_2 implies that high-productivity firms are becoming less reliant on skilled workers. A summary of the changes can be found in Table 6.

As we can see, the model correctly predicts an overall small change in the informality rate. The prediction for unskilled workers is reasonably close to the target, while the model's prediction for an increase in informality among skilled workers is only 20% of the actual change. Overall, the error is around 2 percentage points, and the direction of all changes is correctly estimated. Regarding unemployment, the model is not able to predict the changes accurately, although this could be due to the fact that unemployment in the data shows very small variations. Overall, the errors are around 2 percentage points. In respect to wages, the direction of all changes is correct, although there are some differences in the magnitudes, especially in the case of unskilled workers.

For the formal and size wage premiums, we observe that the predictions face the same issues as in the calibration phase. However, the shares of employment are correctly predicted, except for the case of informal workers in larger firms. The change in the labor share is correctly predicted.

[18]This process is similar to the one in the baseline model by Haanwinckel & Soares (2021), although they only adjust the bias parameters to match changes in wages for both worker types. I incorporate unemployment into the adjustment since the data indicates that skilled workers are increasing their wages but also experiencing higher unemployment, which is challenging for the model to capture.

Table 6: Change in parameters for validation exercise

Parameter	2006	2017
η (share of skilled workers at the national level)	0.2188	0.3775
λ_s^{for} (formal skilled job destruction)	0.0131	0.0152
λ_s^{inf} (informal skilled job destruction)	0.0289	0.0260
λ_u^{for} (formal unskilled job destruction)	0.0149	0.0149
λ_u^{inf} (informal unskilled job destruction)	0.0329	0.0256
w_m (minimum wage)	1.0000	1.3614
A (productivity shifter at the national level)	6.9509	6.7146
τ (payroll tax)	0.6507	0.6273
a_s, a_u (variable benefits)	[0.293; 0.495]	[0.319; 0.504]
m (measure of firms)	0.0790	0.0870
P (enforcement shifter)	-6.3221	-5.0033
α (decreasing returns)	0.4326	0.4846
B_1 (skill bias shifter)	1.4985	0.2547
B_2 (relative skill bias in productive firms)	0.1974	0.1925

5.6 Counterfactuals

Once we have assessed the model fit and validation, it is interesting to see how the key elements of the model would change if we do not take into consideration some of the adjustments. Similar to the previous sections, I will follow the baseline structure set by Haanwinckel & Soares (2021), but with the specific adjustments tailored to the Chilean context.

First, we will examine the changes in the parameters estimated from the data reduction process, including the measure of firms, as shown in Table 7. If we do not consider the change in the fraction of skilled workers, the informality rate would be around 13 percentage points higher. This increase is primarily driven by an increase in the informality rate of unskilled workers, while slightly reducing the informality of skilled workers. In terms of unemployment, there are small changes, while wages would drop on average. This drop is mainly driven by a significant decrease in wages for unskilled workers, although it would imply a significant increase in wages for skilled workers.

If the change in the destruction rates is not considered, we will observe a small increase in the overall informality rate. However, this change will be around 5 percentage points for unskilled workers, indicating that labor stability is a significant component of informality for this worker group. When it comes to unemployment, once again, no relevant changes are observed. Nevertheless, the change in the destruction rates affects wages. It leads to a decrease in wages for skilled workers by around 5%, while increasing the wages of unskilled workers by 8%. This outcome is consistent with how the rates affect the value of each worker type in the formal sector.

Table 7: Conterfactual exercises with parameters estimated from data reduction

		All changes, except		
Outcomes	All changes	Fraction skilled	Destruction rates	Measure of firms
Informality rate	0.3191	0.4539	0.3409	0.3412
Skilled	0.1793	0.1427	0.1612	0.1770
Unskilled	0.3976	0.5337	0.4428	0.4343
Unemployment	0.0929	0.0901	0.0920	0.0967
Skilled	0.1366	0.1519	0.1305	0.1346
Unskilled	0.0663	0.0727	0.0687	0.0738
Wages (ln)	0.7163	0.6699	0.6977	0.5982
Skilled	1.4013	1.8800	1.4550	1.3000
Unskilled	0.3008	-0.0642	0.2383	0.1724

Regarding the measure of firms, if the increase is not considered, we would observe a similar effect to the one generated by the destruction rates. We will see a slight increase in informality, no relevant changes in unemployment, but significant changes in wages. In this case, both skilled and unskilled wages would decrease by 10 and 13 percentage points, respectively.

Second, we will examine the counterfactual effects of the parameters associated with public policy, including productivity, minimum wage, payroll taxes, worker benefits, and government enforcement. Given that payroll taxes and worker benefits are affected by the same structural change (the decrease in the commissions of the pension system), they will be considered jointly. The exercises are shown in Table 8.

Table 8: Conterfactual exercises with parameters associated with public policy decisions

		All changes, except			
Outcomes	All changes	Productivity	Minimum wage	Payroll taxes and benefits	Enforcement
Informality rate	0.3191	0.2930	0.2016	0.3346	0.3795
Skilled	0.1793	0.1732	0.1687	0.1921	0.2260
Unskilled	0.3976	0.3598	0.2194	0.4146	0.4647
Unemployment	0.0929	0.0912	0.0719	0.0930	0.0880
Skilled	0.1366	0.1385	0.1367	0.1363	0.1374
Unskilled	0.0663	0.0625	0.0326	0.0667	0.0580
Wages (ln)	0.7163	0.7779	0.7499	0.6970	0.7337
Skilled	1.4013	1.4578	1.4299	1.3892	1.4142
Unskilled	0.3008	0.3654	0.3373	0.2770	0.3209

As expected, the decrease in productivity observed during the period 2006-2017 resulted in a modest increase in informality, with no significant impact on unemployment. However, it did lead to a reduction in wages of approximately 5% for both skilled and unskilled workers.

Regarding the minimum wage, if it had not been adjusted, we would have observed a decrease in informality of approximately 11 percentage points. This decrease is primarily driven by an

almost 18 percentage point reduction in informality among unskilled workers. The same happens in the unemployment, where the unemployment rate of skilled workers remains the same, while the rate of unskilled workers decrease significantly. Wages would have been positively affected, increasing by approximately 3%. It is relevant noting that the increase in wages resulting from a reduction in the minimum wage implies that the minimum wage significantly affects the optimal ratio of skilled to unskilled workers in firms. Consequently, a reduction in the minimum wage would enable firms to restructure the composition of skilled and unskilled workers, capitalizing on the complementarity benefits of employing both worker types.

In terms of the changes in payroll taxes and benefits resulting from the pension system commission adjustments, we observe a slight decrease in informality for both worker types. This policy did not generate changes in unemployment but did have a slight effect on wages, leading to an increase for both worker types. Lastly, the adjustment in enforcement has a notable impact on the informality rate. The increase in this parameter results in a decrease in informality of approximately 6 percentage points, with a similar effect on both worker types. While this policy proves highly effective in reducing informality, it does not significantly impact unemployment but does lead to a slight increase in wages.

Third and last, we will examine the counterfactual effects of the parameters associated with the adjust in the production function, which includes the decreasing returns and the skilll bias parameters. The exercises are shown in Table 9.

Table 9: Conterfactual exercises with adjustment in production function parameters

		All changes, except		
Outcomes	All changes	Decreasing returns	Skill bias shifter	Relative skill bias
Informality rate	0.3191	0.4847	0.1823	0.3343
Skilled	0.1793	0.2069	0.1705	0.1377
Unskilled	0.3976	0.6464	0.1887	0.4219
Unemployment	0.0929	0.0825	0.0747	0.0934
Skilled	0.1366	0.1057	0.1334	0.1377
Unskilled	0.0663	0.0685	0.0391	0.0666
Wages (ln)	0.7163	-0.2151	0.8731	0.6773
Skilled	1.4013	0.8825	1.2626	1.4120
Unskilled	0.3008	-0.8809	0.6368	0.2613

The impact of the decreasing returns parameter is pivotal. Without the change in the labor share, the informality rate would be approximately 17 percentage points higher, with unskilled workers experiencing an informality rate exceeding 60%. Surprisingly, this parameter has an insignificant effect on unemployment for unskilled workers and actually increases unemployment for skilled workers. This outcome arises because when labor becomes more critical in the production

function, complementarity effects play a more prominent role, relying less on the productivity of each worker group independently. Consequently, after the change in the labor share, firms adjust their skill composition towards a more balanced structure, hiring more unskilled workers and fewer skilled workers. As composition effects become more relevant in this scenario, wages increase significantly for both worker types.

The impact of the skill bias shifter is highly significant as well. The change in this parameter implies that between 2006 and 2017, all firms increased their demand for skilled workers, regardless of their productivity or size. However, this change, which mainly focuses on increasing the demand for the already high-productivity and low-informality group, is not welfare-improving. While the high demand for these workers increases their wages, it doesn't substantially affect their unemployment or informality rates. This result arises because the change in demand is moving in the opposite direction of incentivizing complementarity between both worker types. Additionally, it is essential to remember that skilled workers are relatively scarce, comprising only around a third of the workforce. On the other hand, the effect on unskilled workers is substantial. The change increases their informality and unemployment by around 18 and 3 percentage points, respectively, while decreasing their wages by approximately 33%.

Finally, the change in the relative skill bias generates slight effects on unemployment rates and wages, but it has moderate effects on informality. Specifically, the changes increased the informality rate of skilled workers by around 4 percentage points, while reducing the informality rate of unskilled workers by 2 percentage points.

6 Main differences between Brazil and Chile

In this section, I will discuss the results obtained in the previous section and make an effort to anticipate the effects of the policy changes that have been discussed in the Chilean context. Regarding the first topic, it is clear that there are structural differences between Brazil and Chile that are causing changes in the same direction to generate different effects on informality.

First and foremost, it is essential to acknowledge the differences in tertiary education levels in both countries. While Brazil experienced an increase in tertiary education from 4.4 to 11%, Chile saw a change from 27 to 38%. This suggests that the model exhibits a kind of diminishing returns regarding the effects of increasing the share of skilled workers. To explore this aspect further, I estimated the model for the Chilean calibration with various shares of skilled workers, specifically using shares of 37, 42, 47, 52, 57, and 62% of the workforce. The results indicate that the overall informality rate in these cases is 31.9, 27.6, 22.2, 19.7, 19.3, and 19.1%, respectively. This demonstrates that the model indeed incorporates a logic of diminishing returns to tertiary

education, and in Chile, it operates within this zone. To achieve a 12 percentage point decrease in informality, it is necessary to increase the share of skilled workers by an additional 15 percentage points, and beyond that point, further increases in tertiary education have minimal impact on reducing informality.

Another relevant aspect is the structural difference in the labor markets of both countries. In particular, it's important to highlight the high destruction rate of unskilled jobs in the informal sector in Brazil, which is 8.5%. This rate is more than double the destruction rate in the Brazilian formal sector (3.2%) and almost four times the destruction rate of the Chilean informal sector in both periods (2.9 and 2.6%, respectively). This leads to informal firms in Brazil being highly exposed to turnover and search frictions, making them smaller than in the Chilean case. In contrast, in the Chilean labor market, informal firms offer jobs that are not significantly different in terms of stability from those offered by formal firms. Therefore, when the tertiary education increases, both types of firms can grow in size, leading to a smaller decrease in informality compared to Brazil.

In the same aspect, it is also relevant the differences between the destruction rates of formal jobs for skilled workers. In Brazil the estimated rate is around 0.4% while in Chile the associated rate is close to 1.3% in 2006 and increases to 1.5% in 2017. Then, the formal jobs for skilled workers in Brazil are almost four times more stable than similar jobs in Chile. This significant difference implies that formal firms in Chile are more exposed to search costs than the formal firms in Brazil. This is reinforced when the relative cost of a skilled vacancy in the formal sector is around 3 times more expensive than the formal vacancy for an unskilled worker in Brazil, while in Chile the proportion is around 70 times higher. In constrast, unskilled workers in Chile are relatively cheaper to find since the cost is around just 16% of the cost in Brazil. Nevertheless, the substantial disparities in the relative costs of posting vacancies for skilled and unskilled workers could create disincentives for firms to hire both types. This phenomenon could also contribute to the high unemployment rate experienced by both skilled and unskilled workers in the country.

Regarding Figure 4, it is interesting to notice that in this case, firms around $\log_{10} z = 1$ stop being informal, which is lower than the Brazilian case where they found this phenomenon around $\log_{10} z = 1.5$. In addition, for very low productivity firms ($\log_{10} z < 1$), we can see that the share of informal firms follows a descending stepwise distribution where the first step indicates that the totality of firms in that level of productivity is informal, following of 80%, then around 60%, and so on. The above is not the case in the estimation of Brazil since the first step is around 50%. This implies that Chilean informality is much more concentrated in low-productivity firms than in the Brazilian case.

In addition, it is important to note that some policy components are being utilized similarly

in both countries, while others are moving in opposite directions. For example, in terms of government enforcement, both countries saw an increase in the number of inspections conducted by their respective labor offices. However, Brazil increased its enforcement efforts by 33.9%, whereas Chile's increase was only 20.9%, nearly a third less. These components are essential to bolster since, at worst, they tend to be fiscally neutral (as demonstrated by Almeida & Carneiro (2012) in the case of Brazil, where they generate more revenue than the cost of their operations). Moreover, counterfactual exercises reveal that these components effectively reduce the informality rate without increasing unemployment.

On the other hand, there's the issue of Total Factor Productivity (TFP). While Brazil experienced a 24% increase in TFP during the study period, Chile saw a 3.4% decrease. Consequently, Brazil reduced its informality rate by approximately 4.5 percentage points due to this component, whereas Chile's informality rate increased by 2 points.

Lastly, there are the effects associated with the minimum wage. The case of Brazil is intriguing, as they increased the minimum wage by 61% in real terms over the study period. This substantial increase might have been expected to result in a significant change in the informality rate. However, the counterfactual exercise presented by Haanwinckel & Soares (2021) reveals that the change in the minimum wage only led to a 2-percentage-point increase in informality. This suggests that the minimum wage in Brazil is not binding for a substantial number of firms. In contrast, in the Chilean case, we observe a 36% increase in the real minimum wage, which translates to an 11-percentage-point increase in informality. This highlights that it is not only the magnitude of the minimum wage increase that matters but also how binding it is for formal sector firms.

7 Conclusions

The literature on informality has established a consensus that increasing levels of schooling is an effective means of reducing labor informality. The primary mechanism underlying this is that as educational levels rise, skilled workers tend to join larger firms. This, in turn, due to complementarity relationships in the production function, drives up the demand for unskilled workers, leading to improvements in both their wages and formality status.

Considering the substantial increase in tertiary education in Chile, it becomes interesting to estimate the effects of this increase on the informality rate. However, Chile lacks an official informality series spanning the last three decades. Therefore, the first contribution of this paper is to construct such a series, drawing from both surveys and administrative records. This newly created series is particularly valuable as it demonstrates that the informality rate has remained relatively stable over the observed period.

Next, I adapt the model proposed by Haanwinckel & Soares (2021) to the Chilean context for the period 2006-2017. I chose this model because it incorporates a production function with complementarity across worker types. This feature means that informality does not just decrease due to the level effects of education (where highly educated workers face lower informality rates), but also due to general equilibrium effects (where low-educated workers now face lower informality rates because they are scarcer in the economy). The estimation and counterfactual exercises reveal that the model effectively explains the stability of the informality rate in Chile. This stability is primarily driven by factors such as increases in the minimum wage and the decrease in total factor productivity (TFP), among other components. These factors offset the impact of the rising tertiary education levels.

Next, I delve into the structural disparities between Brazil and Chile. Firstly, I unveil that the influence of tertiary education on informality exhibits diminishing returns, with Chile nearing a point where these effects become negligible. Additionally, I highlight substantial discrepancies in job destruction rates between the two labor markets, largely attributed to the similarities between formal and informal jobs in Chile, while in Brazil, they significantly differ. This divergence is further accentuated by variations in vacancy cost structures, which are relatively uniform in Brazil but exhibit a notable bias towards hiring unskilled workers in Chile. Consequently, achieving an optimal skill composition is considerably more challenging in Chile than in Brazil. Finally, I explore how policies aimed at boosting TFP and enhancing labor law enforcement in Chile can be highly advantageous in reducing informality without adversely affecting unemployment rates.

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8 Appendix

8.1 Compensating differentials, formal wage premium and firm-size wage premium

The model has three sources of cross-firm wage dispersion among similar workers: compensating differentials, minimum wages, and firm-size wage premiums. The first one is associated with the fact that the informal sector has to pay higher wages than the formal sector to compensate for the loss of the mandated benefits (a_i and b_i). It is easy to see this if jobs in both sectors have the same expected duration. Given that, in the data, formal sector jobs have a longer expected duration, the wages in the informal sector should be even higher.

The existence of a binding minimum wage alters the above in some measure and will create a formal wage premium. When the minimum wage is binding, the workers affected by it have a higher *de facto* bargaining power, and therefore formal jobs will be strictly preferred to informal ones since the informal sector cannot fully compensate for this. However, employment in the informal sector will still be possible since it is costly to remain unemployed and wait for a better job offer.

Finally, there will be a firm-size wage premium. This will be generated because of the differences in the replacement costs of workers. Since more productive firms will hire more workers, the rents of the match are higher in larger firms. Given the bargaining process, a share of these rents will go to the workers, who will see higher wages in larger firms. This will generate that the workers will strictly prefer larger firms. Since larger firms will be formal, this is a secondary channel to create the formal wage premium mentioned in the last paragraph.

8.2 Data Reduction: from Age-Education to Skill

It is crucial to address the distinctions between the age and education groups chosen for this study, as they differ significantly from those used in Haanwinckel & Soares (2021). The education groups chosen for this study consist of three categories: workers with primary or incomplete secondary education, those with complete secondary education, and individuals with tertiary education. This classification departs from the one used by the authors, which categorized education levels as 0-7 years, 8-10 years, and 11 or more years of schooling. The original classification posed certain

inconsistencies when applied to the Chilean context. For instance, under the original scheme, the first group encompassed workers with incomplete primary education, the second group included those with complete primary education or the first two years of secondary education, and the last group consisted of individuals with 3 or 4 years of secondary education, in addition to any form of tertiary education. This classification raises confusion because workers with incomplete secondary education may have different probabilities of being considered high-skilled based on whether they are in their second or third year of secondary education. Moreover, a worker with three years of secondary education would share the same probability of being high-skilled as someone with a complete tertiary education. To address these inconsistencies, we have introduced a revised classification that categorizes workers into three distinct groups: primary or incomplete secondary education, complete secondary education, and tertiary education.

Regarding the age groups, Haanwinckel & Soares (2021) categorized individuals into 16-19, 20-24, 25-29, and 30-59 years. The choice to depart from the previous classification is rooted in the understanding that minor variations in age not necessarily imply significant shifts in the likelihood of being classified as high-skilled in the Chilean context. While the authors did not explicitly justify their age groups, their selection aligns with the wage progression patterns observed in Brazil, where early years of work experience often lead to substantial wage increases. However, this wage dynamic does not parallel the situation in Chile, where wage progression tends to be more gradual and consistent over the years (Cases & Vergara, 2020). In light of these differences, the selected age groups in this study aim for a more uniform distribution of age categories, eliminating workers under 18 years old. This omission is justified by the limited participation of children in the Chilean labor market and their ineligibility for unemployment insurance benefits, which renders it impractical to assess the accuracy of informality measures for this age group.

8.3 Imputed parameters: Differences between Brazil and Chile

Upon establishing these parameters, comparing them with the corresponding values in the Brazilian context can unveil critical structural disparities between the two economies. Primarily, when examining the share of skilled workers, we find a striking distinction between Brazil and Chile. In the Brazilian case, this figure hovers around 0.52, whereas in Chile, it stands at 0.22. Notably, the selection of these parameters is designed to minimize differences within each group, and this significant variance in parameter values between the two countries implies substantial disparities in labor income. Specifically, the lower parameter value for Chile suggests that the income gaps within the country are notably wider when compared to the more evenly distributed labor income observed in Brazil.

Another significant aspect to consider is the variation in destruction rates within each skill-

formality group. Broadly speaking, the informality sector in Brazil exhibits considerably higher job insecurity compared to its Chilean counterpart. Conversely, the formal employment sector in Brazil is notably more stable than its Chilean counterpart. For instance, when examining the destruction rate of informal jobs for unskilled workers, it becomes evident that in Brazil, this rate is approximately 0.085, whereas in Chile, it stands at 0.033. Conversely, when considering the destruction rate of formal jobs for skilled workers, Brazil records a rate of around 0.004, while Chile reports a rate of 0.013. These disparities underscore the significant differences in the opportunity cost associated with informal employment between the two countries, with Brazil displaying a substantially higher cost compared to Chile.

When examining the measure of firms in both countries, a noteworthy observation is that Brazil exhibits a higher measure, standing at 0.091 in contrast to Chile's 0.079. This discrepancy signifies that in Brazil, there is a greater number of firms per worker, leading to increased competition for labor resources. All else being equal, this heightened competition tends to drive up wages and reduce informality levels in the Brazilian labor market.